Performance Based Learning and Assessment Task

Trouble in the Orchard

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

The students will use a geometric model and right angle trigonometry to solve a real life problem in several contexts. They will then design a poster or a booklet summarizing the astronomy and trigonometry needed to use shadow lengths to monitor tree growth. Students need to be fluent in organizing and consolidating ideas, presenting these ideas and setting up models to help them understand real life problems.

II. UNIT AUTHOR:

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III. COURSE:

Geometry

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

SOL

G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM

a) Use geometric modeling to solve problems
b) Organize and consolidate their mathematical thinking though communication
c) Analyze properties and determine attributes of three-dimensional objects
d) Use technology

VI. REFERENCE/RESOURCE MATERIALS:

a) One printed copy of each worksheet per student: Think, Pair, Share Sheet; Reason for the Seasons Worksheet
b) Changing Shadows Instructions posted on class website
c) A computer and graphing calculator for each student
d) For each partner set will need: a tennis ball, two push pins, a rubber band, a skewer, a flashlight, masking tape, protractor, and a ball of clay

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment that can be used to guide student work. The teacher will use an adapted form of the assessment as a rubric to evaluate student work. Please find this assessment and rubric attached.

VIII. EVALUATION CRITERIA:

Please see attached assessment and rubric. A benchmark of exemplary can be downloaded as a separate document.

IX. INSTRUCTIONAL TIME:

The activity will take 90 minutes for most students.

Trouble in the Orchard

Strand
Geometry

Mathematical Objective(s)
The students will use right triangle trigonometry to solve a real life problem.

Related SOL

- G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM Standards

- Use geometric modeling to solve problems
- Organize and consolidate their mathematical thinking though communication
- Analyze properties and determine attributes of three-dimensional objects
- Use technology

Additional Objectives for Student Learning:

- ES 1d will be introduced or reinforced depending upon assessment obtained through Think, Pair, Share Activity.

Materials/Resources

- Classroom set of graphing calculators and computers
- One printed copy of each worksheet per student: Think, Pair, Share Sheet; Reason for the Seasons Worksheet
- One printed copy of the assessment sheet and the rubric for each group
- LCD projector for the video clip to be shown during the whole group presentation time
- The Changing Shadows Online Instructions should be copied into a new document and be posted on the class website for student to download.

Assumption of Prior Knowledge
This lesson assumes that the student has completed textbook study of right triangle trigonometry and is operating on Analysis level on Van Hiele scale with respect to the concepts of right triangles and their properties. Students should use the following vocabulary: longitude, latitude, right angle, hypotenuse, opposite, adjacent, summer solstice, winter solstice, fall equinox, spring equinox, earth's axis, hemisphere, north-pole, south-pole. Students may find difficult to understand how region of the earth receive no sunshine during a solstice while other regions are never dark at this time. Students may also have difficulties understanding the relationship between the angle of the sun's rays and the length of the shadow produced.

Introduction: Setting Up the Mathematical Task

In this task, the students will apply their knowledge of right triangles to real life situations. They will use a flashlight and a push pin in investigate the relationship between the angle of a light source and the length of a shadow. They will build a model of the earth on its axis and further investigate how rotation and the earth's tilt of 23.5 degrees impact shadow length. Finally they apply their knowledge to a real life situation and will use the

measure of a given angle and a measure of an adjacent side to predict the length of a shadow on a given date. In this task, the angle is produced by the sun's rays and a tree and the tree it the adjacent side.

The student's time will be broken down in the following manner. The students will spend about 15 minutes on a think, pair share activity, 10 minutes on a whole class lesson, 30 minutes building a model and answering questions related to the model and 30 minutes completing the performance based assessment task. The final 5 minutes will be devoted to a closure activity.

Each pair of students will design a booklet or poster which requires that they understand the impact of latitude and the date on the angle of the sun's rays. They will apply this knowledge and their knowledge of right triangle trigonometry as they put together all the information Uncle Bobby will need to monitor tree growth. Additionally, they will model right triangle trigonometry in GeoGebra.

The task is introduced as students use the attached worksheet and teacher instructions to guide them in a think, pair, share activity which will enable them to draw on prior knowledge and to make their mathematical thinking public. Please see the attached Think, Pair, Share Activity Sheet and the Reason for the Seasons Worksheet and answer sheets for details.

Student Exploration

Whole Class Sharing/Discussion

The class will watch the following video and will change the appropriate think, pair, share answers. The class will then spend 5 minutes discussing their changes and the rationale for these changes.

https://www.youtube.com/watch?v=Pgq0LThW7QA

Small Group Work

The teacher will introduce Small Group Work time by recapping Uncle Bobby's dilemma and sharing the benchmark as one possible solution with the class.

"Three years have gone by since you have heard from your Uncle Bobby. Your orchard is planted as planned and you are enjoying watching your baby trees grow up. All is well until Bobby contacts you to let you know he needs your help again. Several of his trees have contracted leucostoma canker, a fungus that attacks the woody parts of stone fruit trees. His extension agent helped him choose an appropriate treatment and now suggests he monitor the growth of the affected trees four times a year to determine when further treatment is needed.

The agent explained how Bobby could use the length of the tree's shadow and several websites to determine the tree's height (and so avoid having get on a ladder to measure them). Bobby likes the idea of not having to get on the ladder, but he thinks it is crazy that the angle he will use to calculate the tree's height varies with the season. He suspects that for some reason the extension agent may be trying to pull a fast one on him. He admits he doesn't remember much of his high school trigonometry. He wants to know what you think.

You plan to visit Bobby in about a month and want to make a visual aid to help him understand that the extension agent is correct and to help Uncle Bobby remember how to make his calculations. Design a booklet or a poster to help Bobby remember his trigonometry and to guide him as he monitors his trees. His average tree is now about ten feet tall. Bobby

has been instructed to calculate the height of the selected trees on March 20, June 21, and September 22, and December 21. He will then report the results of these measurements to the extension agent."

The teacher will explain that each group will chose a favorite city and a favorite tree to illustrate the information that Uncle Bobby needs to use shadow length to calculate tree height. The students will spend 2 minutes brainstorming names of cities and 2 minutes brainstorming names of trees. The teacher or a helper will list the names on the board.

The students will use the *Reason for the Worksheet* to guide their investigation of the relationship between the angle of a source of light and shadow length. Students will then download the attached *Changing Shadows Online Instructions* from the class website for next portion of the lesson. They will work in groups of two or three.

Student/Teacher Actions:

During this small group time students will be setting up their models, doing research on their computers and then designing their booklets or posters. The teacher will circulate around the room asking questions and responding to questions. It may be helpful to point out the photo of the model if students are having difficulties building it. Initially the teacher will want to make sure the students are making the connection between the model they have built and what they have observed happening to their own shadows. If this is a challenge for the students, drawing the shades and using a large flashlight to project different lengths of one student's shadow may be helpful.

Later the students may struggle as they attempt to find a place on the tennis ball that was remains in the light as the tennis ball rotates. Asking them if they have ever heard of the land of the midnight sun may help them connect to prior learning and lead them to experiment by putting the push pin at the poles.

Finally as they work on their booklet or poster students may have difficulties determining how to combine the angle of the earth's axis and the given latitude of their city into an equation. Using a piece of yarn that travels from the flashlight to the end of the push pin shadow may help the student's visualize the various angles and so set up their equations correctly.

Monitoring Student Responses

Students will communicate their thinking and their new knowledge throughout this unit. The students who begin the unit understanding the ways the earth's tilt and rotation impact our seasons will deepen their knowledge as they communicate the necessary concepts to their classmates during the think, pair and share activity. After the video clip, all students will have an opportunity to refine and solidify their understanding of the necessary concepts through a class discussion.

The questions on the *Reason for the Seasons Worksheet* are designed to help the students make connections between their prior knowledge of both the relationship between shadow length and time of day and their prior knowledge of trigonometry of right triangles with the new and concrete knowledge they gain as they build a model. If groups are having difficulty with these connections the teacher should ask them questions to help them review and consolidate their knowledge.

Students may struggle to get started on their booklet or poster. The teacher should assist them by reminding them of the names of cities and trees placed on the board during the earlier brainstorming session. If none of these suit the students, the teacher can help them narrow down regions of interest and types of trees of interest. The students can then check google maps and Stark Bro's Nursery website for more specific ideas.

Once the students have finished their projects the class will determine which group had the longest shadow on Dec. 21 and which group had the shortest shadow on June 21. The teacher will ask the students to make summary statements about what causes a shadow to be long on December 21 and to be short on June 21 by filling in the following blanks.

Given the same location the _____ will cast the longest shadow. Given trees of the same sized, on December 21 the longest shadow will be cast by the tree that is farthest from _____. Given trees of the same size, on June 21 shortest shadow will be cast by the tree closest to the _____.

As the students are thinking, the teacher will display the following website to model the longest and shortest shadows given by the class:

http://www.learner.org/jnorth/tm/mclass/season_simulator.html

The teacher may need to ask questions to lead the students to the answer below.

Answer: Given the same location the tallest tree will cast the longest shadow. Given trees of the same size, on December 21 the longest shadow will be cast by the tree that is farthest from the Tropic of Capricorn $(a\ latitude\ 23.5°\ S)$. Given trees of the same size, shortest shadow will be cast by the tree closest to the Tropic of Cancer $(a\ latitude\ of\ 23.5°\ N)$.

Assessment List and Benchmarks

Please see attached assessment grid, rubric and benchmark.

Name_____

Date_____

Section_____

Think, Pair, Share Activity

You have 5 minutes to work with your partner to answer the following questions. Please include illustrations if you are able. After 5 minutes each group will share their ideas.

1. What causes the seasons?

2. What is special about the summer solstice, on or about June 21?

3. What is special about the fall equinox, on or about September 22?

Name_____

Date_____

Section_____

Answers for Think, Pair, Share Activity

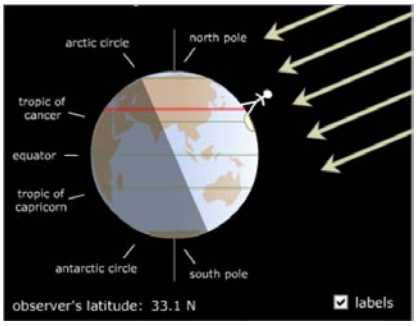
You have 5 minutes to work with your partner to answer the following questions. Please include illustrations if you are able. After 5 minutes each group will share their ideas

1. What causes the seasons?

A: Changes in the angle of the earth's axis in relationship to the sun.

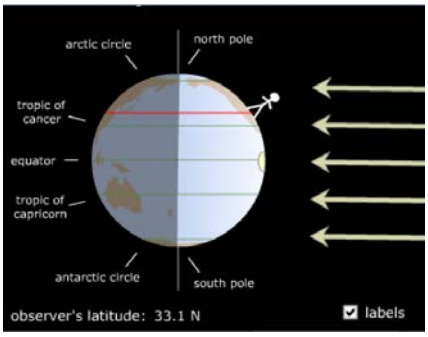
2. What is special about the summer solstice, on or about June 21?

A: At the time of the summer solstice the northern section of the earth's axis is tilted 23.5 degrees towards the sun. In the northern hemisphere it is summer and the longest day of the year. The sun does not set at the north-pole.



3. What is special about the fall equinox, on or about September 22?

A: At the time of the fall equinox the northern section of the earth's axis which had been tilted towards the sun is half way in its rotation to tilt away from the sun. Day and nights are now of equal length.



Name_____

Date_____

Section_____

The Reason for the Seasons Worksheet

Please work with your partner to follow the instructions and to answer the questions below (Use the illustration at the bottom of your sheet to guide your work):

Materials: Tennis ball, rubber band, 2 push pins, wooden skewer, ball of clay, protractor, flashlight, knitting needle (optional)

Procedure Part I:

- Place the rubber band around the center of your tennis ball to represent the equator.
- Push your pin into the tennis ball near, but not on the equator.
- Use your flashlight to produce the smallest possible shadow. Next use your flashlight to produce the longest possible shadow.

Questions #1 to #4 - Please answer in complete sentences:

1. Describe the angle produced by the flashlight and the imaginary plane where the push pin is "standing" when the shadow is the shortest.

2. Describe the angle produced by the flashlight and the imaginary plane where the push pin is "standing" when the shadow is the longest.

3. What time of day will your body produce the shortest shadow? Why?

4. What time of day will your body produce the longest shadow? Why?

Procedure Part II

A. Use your push pin to make a small hole at the north and south-pole of your tennis ball. Run your skewer through these two holes. (You may need a knitting needle to enlarge the hole before the skewer will go through).

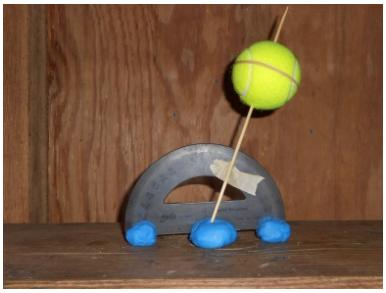
B. Tilt the skewer 23.5 degrees towards the flashlight. This will be 113.5 degrees on your protractor. Use the ball of clay and masking tape to hold the skewer in place. Place the flashlight in front of the 180 degree side of the protractor (so the protractor does not produce a shadow) shine the flashlight on the rubber band and rotate the tennis ball once. Try to place your push pin on a spot where it will always be in the light as the ball rotates. Rotate it again to see if you are correct. Use another push pin and find a spot where it will not be touched by the light of the flashlight as it rotates. Once again check your work with a new rotation.

Questions #5 and #6

5. Describe the position of the push pin that was always in the light. If the ball represents the earth and the flashlight the sun, where would the push pin be and what would you guess about the date?

6. Describe the position of the push pin that was never in the light. If the ball represents the earth and the flashlight the sun, where would the push pin be and what would you guess about the date?

Your finished project should look similar to the project below.



The Reason for the Seasons Answer Sheet

Please work with your partner to follow the instructions and to answer the questions below (Use the illustration at the bottom of your sheet to guide your work):

Materials: Tennis ball, rubber band, 2 push pins, wooden skewer, ball of clay, protractor, flashlight, knitting needle (optional)

Procedure Part I:

- Place the rubber band around the center of your tennis ball to represent the equator.
- Push your pin into the tennis ball near, but not on the equator.
- Use your flashlight to produce the smallest possible shadow. Next use your flashlight to produce the longest possible shadow.

Questions #1 to #4 - Please answer in complete sentences:

1. Describe the angle produced by the flashlight and the imaginary plane where the push pin is "standing" when the shadow is the shortest.

A: The angle produced by the flashlight and plane is 90 degrees.

2. Describe the angle produced by the flashlight and the imaginary plane where the push pin is "standing" when the shadow is the longest.

A: The angle produced by the flashlight and plane is about 10 degrees.

3. What time of day will your body produce the shortest shadow? Why?

A: The shortest shadow will be produced at noon because the angle produced by the sun's rays and my body is small.

4. What time of day will your body produce the longest shadow? Why?

A: The longest shadow will be produced around sunrise or sunset because the angle produced by the sun's rays and may body is large..

Procedure Part II

A. Use your push pin to make a small hole at the north and south-pole of your tennis ball. Run your skewer through these two holes. (You may need a knitting needle to enlarge the hole before the skewer will go through).

B. Tilt the skewer 23.5 degrees towards the flashlight. This will be 113.5 degrees on your protractor. Use the ball of clay and masking tape to hold the skewer in place. Place the flashlight in front to the 180 degree side of the protractor (so the protractor does not produce a shadow) shine the flashlight on the rubber band and rotate the tennis ball once. Try to place your push pin on a spot where it will always be in the light as the ball rotates. Rotate it again to see if you are correct. Use another push pine and find a spot where it will not be touched by the light of the flashlight as it rotates. Once again check your work with a new rotation.

Questions #5 and #6

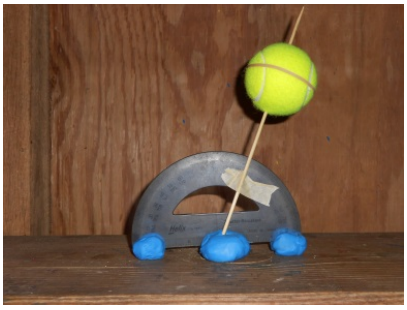
5. Describe the position of the push pin that was always in the light. If the ball represents the earth, the flashlight the sun, and the pushpin a person where would the person be and what would you guess about the date?

A: The person would be at the north-pole and it would be around the time of the summer solstice.

6. Describe the position of the push pin that was never in the light. If the ball represents the earth, the flashlight the sun, and the push pin a person, where would the person be and what would you guess about the date?

A: The person would be at the south-pole experiencing the Southern Hemisphere's winter solstice.

Your finished project should look similar to the project below.



Online Instructions for Changing Shadows

Here is the background for your project:

Three years have gone by since you have heard from your Uncle Bobby. Your orchard is planted as planned and you are enjoying watching your baby trees grow up. All is well until Bobby contacts you to let you know he needs your help again. Several of his trees have contracted leucostoma canker, a fungus that attacks the woody parts of stone fruit trees. His extension agent helped him choose an appropriate treatment and now suggests he monitor the growth of the affected trees four times a year to determine when further treatment is needed.

The agent explained how Bobby could use the length of the tree's shadow and several websites to determine the tree's height (and so avoid having get on a ladder to measure them). Bobby likes the idea of not having to get on the ladder, but he thinks it is crazy that the angle he will use to calculate the tree's height varies with the season. He suspects that for some reason the extension agent may be trying to pull a fast one on him. He admits he doesn't remember much of his high school trigonometry. He wants to know what you think.

You plan to visit Bobby in about a month and want to make a visual aid to help him understand that the extension agent is correct and to help Uncle Bobby remember how to make his calculations. Design a booklet or a poster to help Bobby remember his trigonometry and to guide him as he monitors his trees. His average tree is now about ten feet tall. Bobby has been instructed to calculate the height of the selected trees on March 20, June 21, and September 22, and December 21. He will then report the results of these measurements to the extension agent.

Your poster or booklet should contain the following six components. You may hand draw your project or use images you find online. Helpful websites are given with the components when appropriate.

1. Title- Pick a favorite tree and a favorite city. The city can be anywhere in the world as long as the latitude is not $33°$. Remember that if you chose a city in the southern hemisphere, summer solstice will be in December and winter solstice will be in June. Fill in the following blanks to create your title:
   Shadow of a _____ Foot _____Tree in _____. . Next use this title to make a title page or to write the title across the top of your poster.

2. The equation needed to find the tangent in right angle geometry.

   https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--using-soh-cah-toa

3. An illustration of: $tangent = \frac{opposite}{adjacent}$

   https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle

4. Use the website below to help you understand the angle of the sun's rays on each of the dates given by the extension agent: June 21, September 22, December 21 and March 20.

   http://www.learner.org/jnorth/tm/mclass/season_simulator.html

http://www.uic.edu/classes/phys/phys112ma/lab_latitude.htm

5. The equation for the angle that the sun's rays produce on each of these dates.

6. A right triangle illustrating the general shape of the shadow triangle for each date. Use your protractor or GeoGebra to produce these triangles.

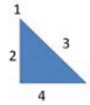
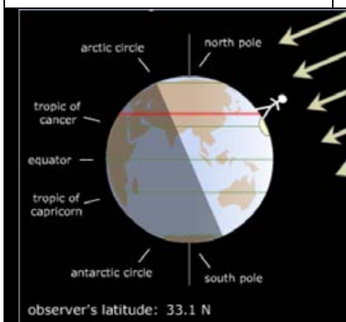
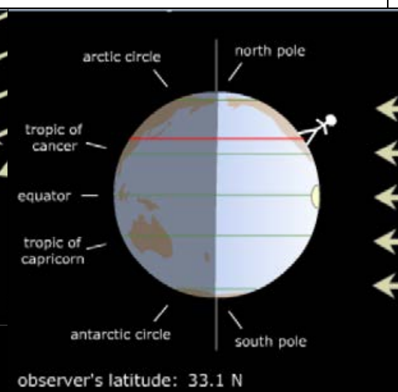
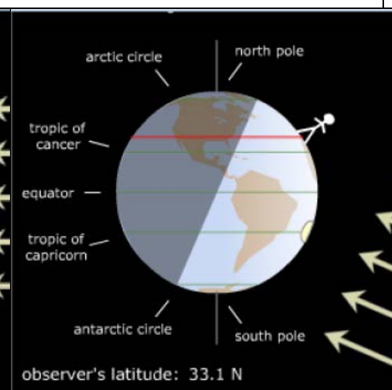
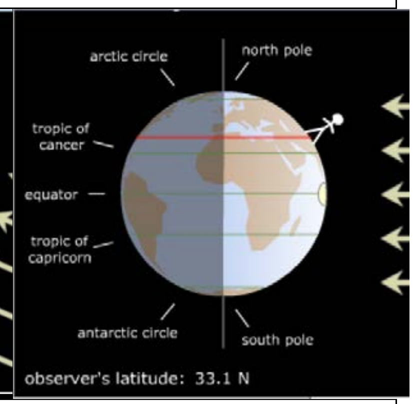
Please download the benchmark for this task if you would like a closer look at what is expected.

Trouble in the Orchard Assessment List

			Earned Assessment	
Num.	**Element**	**Point Value**	**Self**	**Teacher**
1	The poster or booklet conveys a deep understanding of the necessary concepts	2		
2	The shape of the tree, shadow, hypotenuse triangle is correct for each date	2		
3	Mathematical formulas, illustrations and calculations are correct	2		
4	Work is neat and attractive.	2		
5	The poster or booklet is complete	2		
6	The poster or booklet clearly conveys the needed information	2		

Rubric for Trouble in the Orchard

#	Element	0	1	2
1	The poster or booklet conveys a deep understanding of the necessary concepts	The poster or booklet conveys no understanding of the necessary concepts	The poster or booklet conveys some understanding of the necessary concepts	The poster or booklet conveys a deep understanding of the necessary concepts
2	The shape and part identification of the tree, shadow, hypotenuse triangle is correct for each date	The student did not label the parts correctly and did not produce the correct shapes	The student correctly labelled the parts or correctly produced the shapes	The student correctly labelled the parts and correctly produced the shapes
3	Mathematical formulas, illustrations, and calculations are correct	None of the Mathematical formulas, illustrations, and calculations are correct	Some of the Mathematical formulas, illustrations, and calculations are correct	All of the Mathematical formulas, illustrations, and calculations are correct
4	Work is neat and attractive.	None of the Work is neat and attractive.	Some of the Work is neat and attractive.	All of the Work is neat and attractive.
5	The poster or booklet is complete	Many parts are missing from the poster or booklet	Some parts are missing from the poster or booklet	The poster or booklet is complete
6	The poster or booklet clearly conveys the needed information	The poster or booklet does not convey information	The poster or booklet conveys some of the needed information	The poster or booklet clearly conveys the needed information

A Review of Trigonometry soh cah toa	$Tangent = \frac{opposite}{adjacent}$	 1. Given angle 2. Adjacent Side	3. Hypotenuse 4Opposite Side
Summer Solstice June 21, 2014	Fall Equinox September 22, 2014	Winter Solstice December 21, 2014	Spring Equinox March 20, 2015
The north pole is tilting towards the sun and so the sun's angle over Georgia is Georgia's latitude minus the tilt. $33° - 23.5° = 9.5°$	The earth is straight on its axis and so the sun's angle at the equator is 90 degrees. The angle over Georgia is 33 degrees.	The north pole is tilting away from the sun. The sun's angle over Georgia is Georgia's latitude plus the tilt of the earth. $33° + 23.5° = 56.5°$	The earth is straight on its axis and so the sun's angle at the equator is 90 degrees. The angle over Georgia is 33 degrees.
			
Summer Shadow of a 10' tree • $9.5°\ Angle$ • $1.7'\ Shadow$	Fall Shadow of a 10' tree • $33°\ Angle$ • $6.5'\ Shadow$	Winter Shadow of a 10' tree • $56.5°\ Angle$ • $15'\ Shadow$	Spring Shadow of a 10' tree • $33°\ Angle$ • $6.5'\ Shadow$